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# An orthogonal multi-input integration system to control gene expression in *Escherichia coli*

## Supplementary Information

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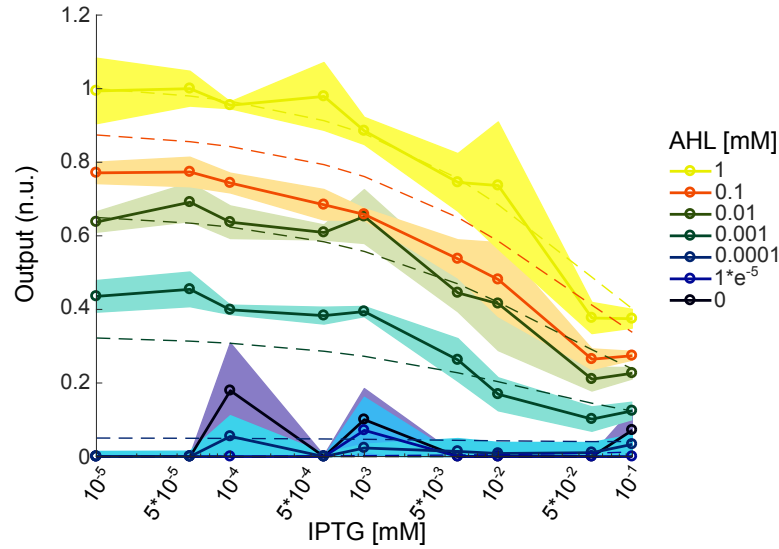
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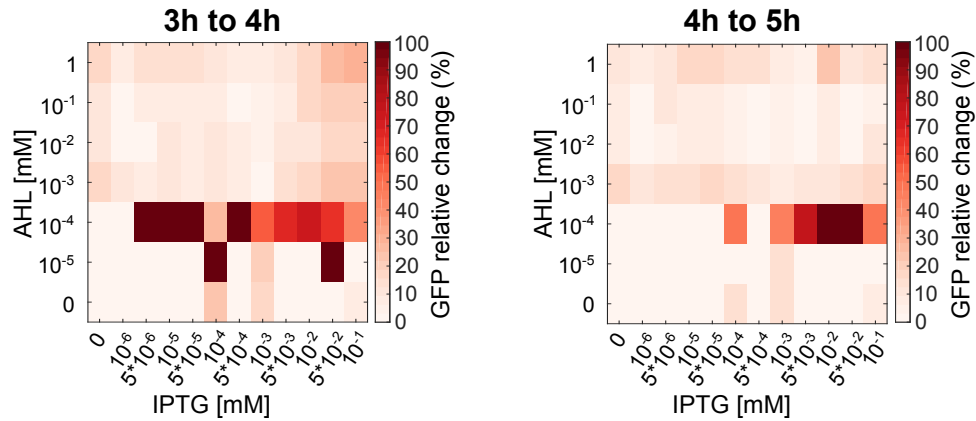
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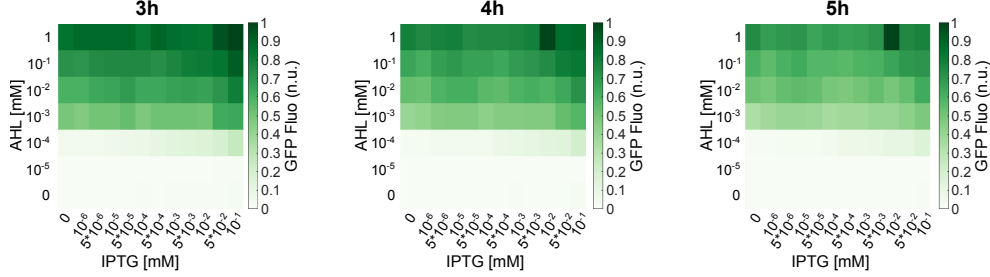
**A)**



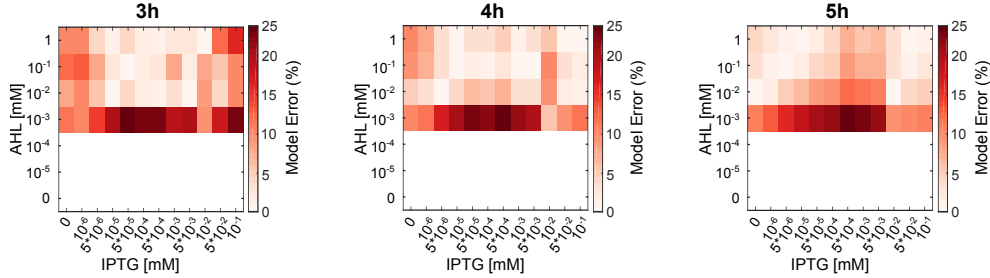
**B)**



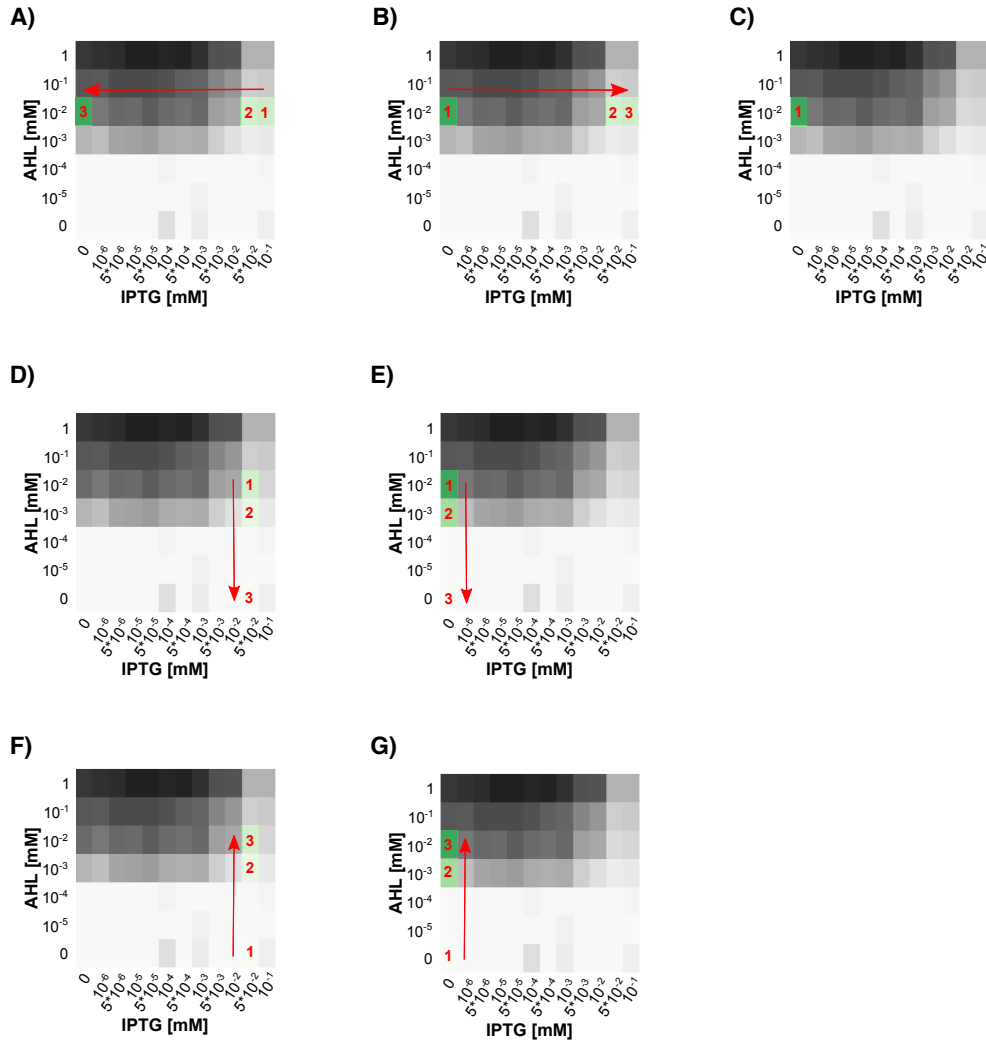
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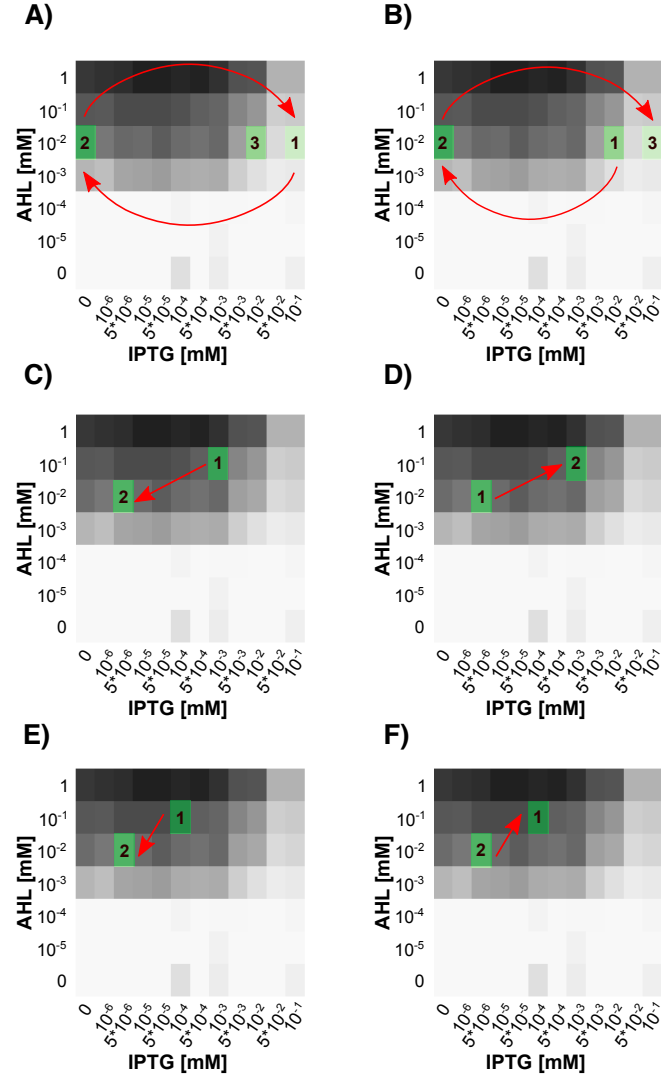
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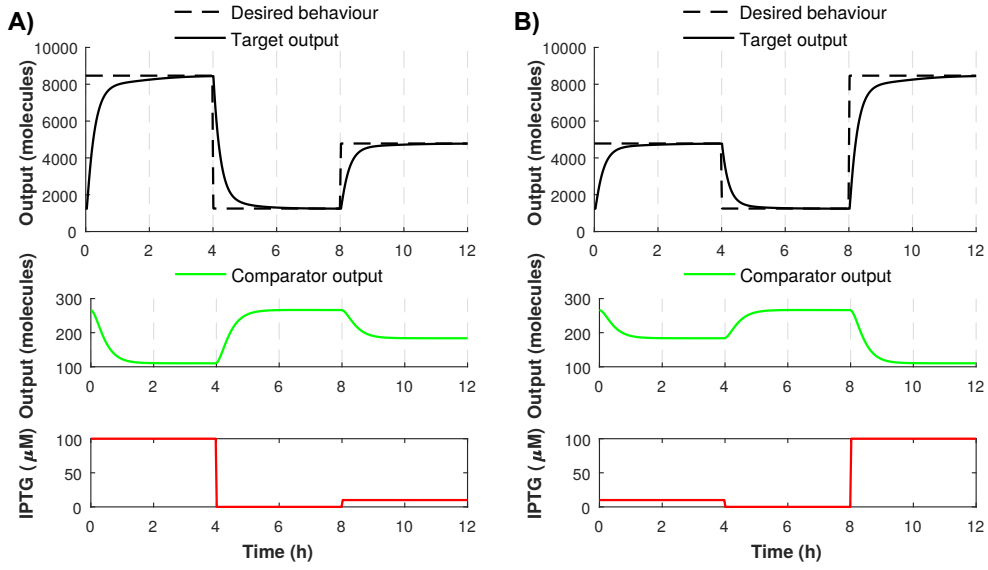
Supplementary Figure S3: Model simulation steady-state error, relative to wide-range characterisation data. Percentage absolute error between the model predictions (Figure 1-C) and the wide-range characterisation (Figure 1-B) are plotted at the indicated time points as heat maps. Data from the wide-range characterisation were filtered as described in Section S4 prior to computation of the error, so that the model predictions were directly comparable to the data used for parameter identification. Data points for AHL  $\leq 10^{-4}$  mM were set to zero for the parameter identification: the (normalised) error was not computed for these values of AHL input, since it would have been infinitely large, and consequently this region is coloured white.



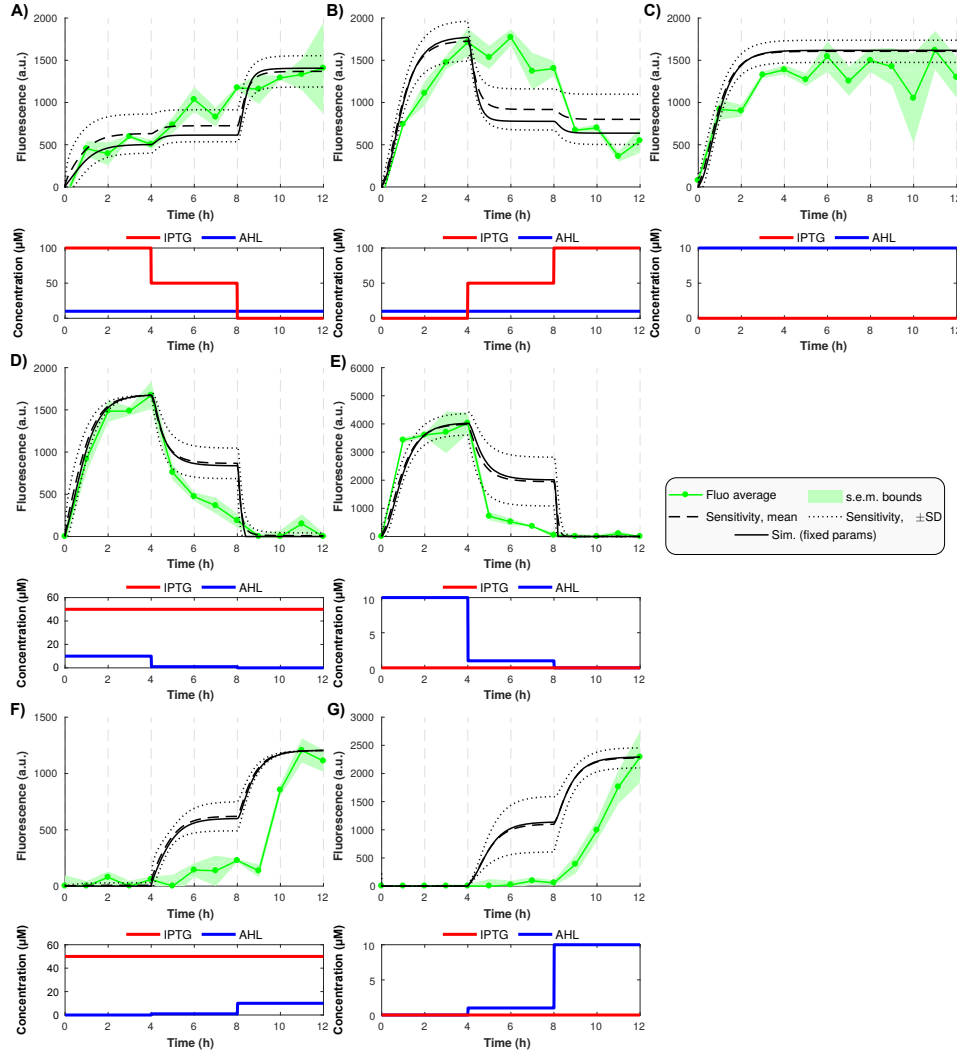
Supplementary Figure S4: Combination of concentrations used in experiments from Figure 2 (Main Matter). **A–G)** Combination of concentrations used in experiment from Figure 2 (Main Matter) mapped on the 4h heatmap from Figure 1 (Main Matter).



Supplementary Figure S5: Combination of concentrations used in experiments from Figure 3 (Main Matter). **A–F)** Combination of concentrations used in experiment from Figure 3 (Main Matter) mapped on the 4h heatmap from Figure 1 (Main Matter).

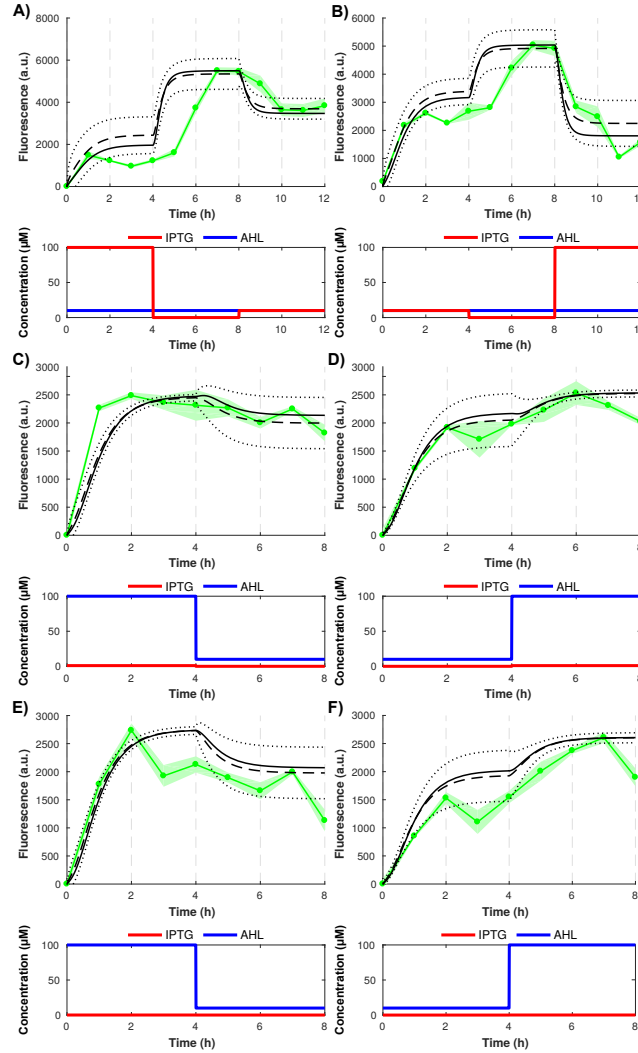


Supplementary Figure S6: Further *in-silico* experiments of the coupled comparator-target consortium. **A)** and **B)** show *in-silico* experiments our proposed controller consortium (Figure 4), using the IPTG input signals shown in Figure 3A and Figure 3B. The desired multi-step output is plotted in the top panels as a dashed line; the actual target output tracking this desired signal, as controlled by the computation module, is plotted as a solid black line. The central panel indicates the actual comparator output over time (green). Finally, the lower panel shows that actual IPTG reference signals corresponding to those in Figure 3A (**A**), and Figure 3B (**B**), that are fed to the comparator module in order to signal the desired response.



Supplementary Figure S7: Computational module model sensitivity analysis corresponding to input signals shown in Figure 2. In each panel, the time-lapse data are plotted as green lines, with sampling points indicated by circles, and the filled green region indicating corresponding s.e.m. over the experiments. Solid black lines indicate the baseline model output, using our final version of the fitted parameters (Table S6), as plotted in Figure 2. The dashed lines show the mean of 7000 Monte-Carlo simulations (per each condition), with each simulation using a perturbed set of parameters; surrounding dotted lines indicate the corresponding region of  $\pm$ SD. In each realisation of the Monte-Carlo simulations, the seven parameters that we had optimised by fitting to wide-range characterisation data were perturbed and the remaining parameters were kept constant. Perturbed parameters' values were chosen from a Normal distribution centred on the fitted value (Table S6) with standard deviation of 20% of that value.





Supplementary Figure S8: Computational module model sensitivity analysis corresponding to input signals shown in Figure 3. In each panel, the time-lapse data are plotted as green lines, with sampling points indicated by circles, and the filled green region indicating corresponding s.e.m. over the experiments. Solid black lines indicate the baseline model output, using our final version of the fitted parameters (Table S6), as plotted in Figure 3. The dashed lines show the mean of 7000 Monte-Carlo simulations (per each condition), with each simulation using a perturbed set of parameters; surrounding dotted lines indicate the corresponding region of  $\pm$ SD. In each realisation of the Monte-Carlo simulations, the seven parameters that we had optimised by fitting to wide-range characterisation data were perturbed and the remaining parameters were kept constant. Perturbed parameters' values were chosen from a Normal distribution centred on the fitted value (Table S6) with standard deviation of 20% of that value.

Plasmid	Promoter/genes	Replication origin	Selection	Source
pLuSb	<b>Nter</b> -6xHis-HRV-3Csite-HMNETDP(linker)- $\sigma$ 20_992-AANDENYALAA(ssrA)- <b>Cter</b> expressed from the lux promoter; LuxR expressed from a constitutive promoter	p15A	Chloramphenicol	This study
pLacASb-Flag	<b>Nter</b> -Anti- $\sigma$ 20_992-AANDENYALAA(ssrA)-Flag- <b>Cter</b> expressed from the lacUV5 promoter; LacI expressed from the lacI promoter	pBR322	Ampicillin	This study
pVRb-ssrA	<b>Nter</b> -sfGFP-AANDENYALAA(ssrA)- <b>Cter</b> expressed from the 20_992 promoter ( $\sigma$ 20_992 responsive)	pSC101	Kanamycin	This study

Supplementary Table S1: Plasmids used in this study.

3h

Means		IPTG											
		0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM
AHL	0 nM	0.0	0.0	0.0	0.0	0.0	28983.1	0.0	15303.7	0.0	0.0	0.0	9751.1
	10 nM	0.0	0.0	0.0	0.0	0.0	1026.6	0.0	11101.7	0.0	0.0	17.6	0.0
	100 nM	0.0	0.0	293.7	474.6	677.2	9300.2	1797.5	6343.4	5627.4	4190.6	3855.7	7139.3
	1000 nM	54708.4	47243.1	58014.6	61731.9	61868.6	56108.7	52455.3	49046.6	38123.6	26158.2	16676.8	20813.9
	10 uM	90574.9	74233.9	80222.7	89364.7	95957.8	90064.5	82142.3	78149.1	62128.1	58742.2	32567.1	33928.0
	100 uM	101364.0	89709.1	105300.3	104549.3	104964.5	100579.3	87586.2	85914.9	73552.1	73138.4	41226.6	43010.0
	1000 uM	128450.1	121477.8	132212.8	147044.3	146186.6	136736.7	134099.0	120736.2	104212.1	114062.1	65283.8	67441.8

4h

Means		IPTG												
		0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM	
AHL	0 nM	0.0	0.0	0.0	0.0	0.0	0.0	22578.5	0.0	12646.2	0.0	0.0	0.0	9035.2
	10 nM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8994.1	0.0	0.0	0.0	0.0	0.0
	100 nM	0.0	0.0	0.0	0.0	0.0	0.0	6916.2	0.0	2937.0	1760.0	1060.6	1341.6	4194.4
	1000 nM	45075.8	41762.1	52974.9	54892.4	57302.2	50281.1	48297.1	49688.2	33094.6	21345.0	12840.9	15657.5	0.0
	10 uM	80229.7	74040.4	81562.3	80367.4	87154.6	80308.5	76784.7	82243.9	56087.3	52392.0	26532.5	28548.7	0.0
	100 uM	91004.6	88770.5	96525.2	97212.3	97516.6	93729.0	86281.0	82967.5	67726.1	60644.7	33363.5	34550.2	0.0
	1000 uM	107618.5	111163.0	114218.2	125281.6	126065.3	120370.7	123413.7	111590.9	93942.4	92865.0	47482.5	47222.0	0.0

5h

Means		IPTG											
		0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM
AHL	0 nM	0.0	0.0	0.0	0.0	0.0	0.0	19095.2	0.0	11030.7	0.0	0.0	8325.2
	10 nM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7761.2	0.0	0.0	0.0	0.0
	100 nM	0.0	0.0	0.0	0.0	0.0	3573.0	0.0	1576.3	410.4	0.0	0.0	2204.5
	1000 nM	37196.9	36651.6	45365.0	46586.7	47838.0	43228.4	43388.2	45574.8	29116.6	18759.0	10975.1	12943.0
	10 uM	73966.2	73557.3	78741.3	74207.5	79469.5	76909.8	74585.6	85943.9	56559.2	49079.5	26880.7	25831.1
	100 uM	82650.0	86424.1	87061.4	89079.2	89368.4	89274.5	88438.5	86787.0	69196.1	55066.2	34030.6	33353.4
	1000 uM	96657.8	102958.8	100793.1	103398.7	102888.2	104714.4	107165.5	101869.7	88877.0	71975.1	42738.8	40117.2

SEMs		IPTG											
0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM		
316.4	751.6	181.3	319.3	75.5	19097.5	806.2	11703.2	996.8	410.6	437.9	8315.7		
206.9	465.2	292.2	107.4	116.7	5754.9	620.5	12149.2	544.6	597.4	4843.9	928.3		
2581.3	1024.1	2993.3	2559.7	2176.7	7604.5	2287.2	2034.3	5827.0	5154.3	3713.0	4445.5		
7775.0	7372.3	5024.8	6664.9	6722.5	4074.0	3769.5	4666.3	9004.1	8107.7	6593.9	4235.1		
12315.6	6817.6	6519.9	2944.0	7334.8	7078.6	4984.8	10899.6	8538.0	19681.0	6905.8	4460.2		
4902.2	10899.3	3424.3	2261.5	3351.8	6440.0	9148.3	8689.8	7494.6	20406.8	6294.5	3763.7		
9072.1	12397.8	6864.7	10185.6	5645.2	4338.4	16975.9	9759.9	11794.6	28292.2	7756.1	7084.4		

SEMs		IPTG											
0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM		
362.4	487.0	355.3	340.0	448.5	16344.6	365.2	10651.5	266.6	672.1	675.6	7181.0		
316.1	294.9	57.4	186.6	225.1	4550.0	26.2	11317.4	127.5	692.7	3704.3	358.5		
1504.7	519.8	2065.3	1676.7	1856.4	7101.3	1203.2	1285.6	4404.2	3797.5	3476.7	3276.9		
4984.7	4111.8	4754.5	5427.3	5937.9	1607.5	2890.9	1749.1	7321.9	5655.3	4174.7	2982.5		
8392.1	1945.2	5331.0	3508.3	6649.2	5585.0	2578.9	9226.5	6484.3	16070.4	4067.7	1964.7		
5711.4	4771.0	4965.7	3661.3	5062.7	3385.8	5230.0	2046.0	6546.4	12677.8	3527.2	1515.0		
10874.6	5101.8	7440.3	11235.1	5975.8	1174.0	11607.3	4650.9	9865.7	21785.0	5334.3	3214.3		

SEMs		IPTG											
0	1 nM	5 nM	10 nM	50 nM	100 nM	500 nM	1 uM	5 uM	10 uM	50 uM	100 uM		
385.9	492.8	318.5	403.1	390.2	15575.9	547.7	9022.4	367.1	320.8	443.0	6600.7		
379.1	420.0	294.2	218.7	181.2	2677.7	357.9	10288.2	257.2	430.9	2312.8	562.3		
914.8	324.0	1241.4	960.9	970.4	4584.9	1004.0	938.0	3343.7	2720.1	2325.7	2911.2		
1859.3	1325.9	2558.8	3433.0	2343.8	2049.3	4128.5	1382.9	5415.8	3130.9	2528.6	1792.8		
1837.4	1873.6	3563.2	1747.0	2016.3	1716.4	3226.8	8455.1	3933.4	9931.3	3058.0	107.3		
1457.5	1583.8	2002.4	4186.3	3325.5	2139.8	3518.8	1856.0	3943.3	4222.5	832.4	217.3		
4674.5	848.8	2925.2	3232.0	3869.4	5354.7	5398.5	1388.5	1912.8	4869.9	526.9	270.7		

Supplementary Table S2: Absolute values and S.E.M. for the wide range characterization experiments. Absolute values and S.E.M. for the wide range characterization experiments. Data normalized by subtracting the background intensity and normalizing on the OD600 measurements. After normalization, data from three independent experiments from each time point are averaged and scaled across the full dynamical range ((measured value - min value)/(max value - min value)).

Name	Sequence
Sigma_FW	5'-ATAGGATTAATCCATGGGCAGCAGCCATCATCATCATCACAGCAGCGG-3'
Sigma_ssrA_RW	5'-ATTTAAGGATCCTCACGCTGCAAGGGCGTAATTTTCGTCGTTGCTGCACTAGTCGGTTTGCGACGA CCGCTCAGATCTGCACC-3'
LacIASb_Vector_F	5'-AGAAGATTTTCAGCCCACCACCACCCTGAGATCC-3'
LacIASb_Vector_R	5'-TGCCTAATGGAATTCGGGATCGAGATCTCGATCCT-3'
LacIASb_Frag1_F	5'-CGAGATCTCGATCCCGAATTCCATTAGGCATTAGGCACCC-3'
LacIASb_Frag1_R	5'-TTCCGGTGTGCCCATCTTTACCTCCTCTATCGCGGA-3'
LacIASb_Frag2_F	5'-ATAGAGGAGGTAAAGATGGGCACACCGGAACGT-3'
LacIASb_Frag2_R	5'-TCAGTGGTGGTGGTGGGCTGAAAATCTTCTCTCATCCGC-3'
ASb_Flag_F	5'-GACGACGATAAGATCGATTACAAGGATGACGACGATAAGTAAGGATCCAAGCTTGGCTGTTTTGGCGG-3'
ASb_Flag_R	5'-ATCCTTGTAATCTCCCTTATCGTCGTCATCCTTGTAATCCGCTGCAAGGGCGTAATTTTCGTC-3'
GFPssrA_F	5'-CTTGCAGCGTAATCCAGACCTGCAGGCATGCAAGCCTCTAGAG-3'
GFPssrA_R	5'-GGCGTAATTTTCGTCGTTGCTGCTTTGTAGAGCTCATCCATGCC-3'
AntiSigma_FW	5'-TTATTCCATGGGCACACCGGAACGTTTTGTTTCATCTGGCAGATGCC-3'
AntiSigma_ssrA_RW	5'-TAGCCAAGCTTGGATCCTTACGCTGCAAGGGCGTAATTTTCGTCGTTGCTGCACTAGTCTGTTC TGCTTCTTCTGCATTAATGC-3'

Supplementary Table S3: Primers used in this study.

## S2 Model Derivation

In this section we describe the features and derivation of the ODE-based mathematical model of our proposed signal computation system GRN (Figure 1-a).

### S2.1 Model assumptions

- We use a continuous and deterministic ordinary differential equation (ODE) model to represent the system dynamics. Reactions are represented through mass-action kinetics, and we do not explicitly account for population growth since the number of cells is kept constant throughout experiments.
- For all species in the system, we assume that the variation in plasmid copy number is not large enough to significantly affect the dynamics and can be neglected from the model.
- Diffusion of IPTG into the population is assumed to occur instantaneously (the internal concentration instantaneously reaches the desired signal reference concentration within the population when modified externally)<sup>12</sup>.
- Similarly we assume the internal AHL concentration instantaneously assumes the externally applied concentration<sup>13</sup>.
- For the AHL, we assume that there is a steady equilibrium concentration of the relevant proteins (LuxR for the activation of AHL), that is sufficiently high to consider any AHL in the cell to be bound as the LuxR:AHL complex and thus act directly on the AHL responsive promoter<sup>2</sup>. Possible aggregated effects of saturation and competition are subsumed under the (fitted) co-operativity coefficient of this promoter (see Section S4).
- We assume that interactions between sigma and RNA polymerase can be neglected, and therefore in the model the p20\_992 promoter is driven directly by the concentration of free sigma. In other words, ancillary dynamics of the sigma factor (e.g., possible competition with endogenous sigma factors, see<sup>9</sup>) are aggregated in the parameters and equation of the p20\_992 promoter governing transcription of GFP (Equation S20).

## S2.2 Summary of dynamical variables in the full model

State variable	Description
$[M_\sigma]$	mRNA concentration for sigma factor.
$[M_{\sigma_\alpha}]$	mRNA concentration for anti-sigma.
$[M_{\text{GFP}}]$	mRNA concentration for GFP.
$[P_{\text{GFP}}]$	Mature GFP concentration.
$[\sigma_{\text{free}}]$	Concentration of free sigma factor.
$[\sigma_{\alpha,\text{free}}]$	Concentration of free anti-sigma.
$[\sigma:\sigma_\alpha]$	Concentration of the sigma:anti-sigma complex (bound sigma and anti-sigma).

Supplementary Table S4: Overview of the seven dependent variables defined in the full model. Here, for  $\sigma$  and  $\sigma_\alpha$ , the subscript ‘free’ denotes the fact that these species are not in complex *with each other* (denoted  $\sigma:\sigma_\alpha$ ); it is unrelated to binding with RNA polymerase, which is not considered in our model (see Section S2.1, ‘Model assumptions’).

## S2.3 Reactions considered in the full model

Reaction	Description
$\xrightarrow{H^+([A])} M_\sigma$	Transcription of sigma ( $\sigma$ ) mRNA.
$\xrightarrow{H^+([I])} M_{\sigma_\alpha}$	Transcription of anti-sigma ( $\sigma_\alpha$ ) mRNA.
$\xrightarrow{H^+([\sigma])} M_{\text{GFP}}$	Transcription of GFP mRNA.
$M_\sigma \xrightarrow{\gamma_{M_\sigma}} \emptyset$	Degradation of $\sigma$ mRNA.
$M_{\sigma_\alpha} \xrightarrow{\gamma_{M_{\sigma_\alpha}}} \emptyset$	Degradation of $\sigma_\alpha$ mRNA.
$M_{\text{GFP}} \xrightarrow{\gamma_{M_{\text{GFP}}}} \emptyset$	Degradation of GFP mRNA.
$M_\sigma \xrightarrow{k_\sigma} \sigma_{\text{free}}$	Translation of $\sigma$ from its mRNA.
$M_{\sigma_\alpha} \xrightarrow{k_{\sigma_\alpha}} \sigma_{\alpha,\text{free}}$	Translation of $\sigma_\alpha$ from its mRNA.
$M_{\text{GFP}} \xrightarrow{k_{\text{GFP}}} P_{\text{GFP}}$	Translation and maturation of GFP.
$\sigma_{\text{free}} \xrightarrow{f(X)} \emptyset$	Degradation of $\sigma$ (ssrA tagged).
$\sigma_{\alpha,\text{free}} \xrightarrow{f(X)} \emptyset$	Degradation of $\sigma_\alpha$ (ssrA tagged).
$P_{\text{GFP}} \xrightarrow{f(X)} \emptyset$	Degradation of mature GFP (ssrA tagged).
$\sigma_{\text{free}} \xrightarrow{\gamma} \emptyset$	Dilution of $\sigma$ due to cell division.
$\sigma_{\alpha,\text{free}} \xrightarrow{\gamma} \emptyset$	Dilution of $\sigma_\alpha$ due to cell division.
$P_{\text{GFP}} \xrightarrow{\gamma} \emptyset$	Dilution of mature GFP due to cell division.
$\sigma_{\text{free}} + \sigma_{\alpha,\text{free}} \xrightarrow{k_{\sigma:\sigma_\alpha}^+} \sigma:\sigma_\alpha$	$\sigma:\sigma_\alpha$ complex formation from one $\sigma$ and one $\sigma_\alpha$ .
$\sigma:\sigma_\alpha \xrightarrow{k_{\sigma:\sigma_\alpha}^-} \sigma_{\text{free}} + \sigma_{\alpha,\text{free}}$	Dissociation of one $\sigma:\sigma_\alpha$ complex into one $\sigma$ and one $\sigma_\alpha$ .
$\sigma:\sigma_\alpha \xrightarrow{f(X)} \emptyset$	Degradation of one $\sigma:\sigma_\alpha$ complex.
$\sigma:\sigma_\alpha \xrightarrow{\gamma} \emptyset$	Dilution of $\sigma:\sigma_\alpha$ complex due to cell division.
$\sigma_{\text{total}} = \sigma_{\text{free}} + \sigma:\sigma_\alpha$	Conservation of $\sigma$ : total sigma is the sum of concentrations in its free and bound forms.
$\sigma_{\alpha,\text{total}} = \sigma_{\alpha,\text{free}} + \sigma:\sigma_\alpha$	Conservation of $\sigma_\alpha$ : total anti-sigma is the sum of concentrations in its free and bound forms.

Supplementary Table S5: A summary of all major reactions considered in our full model of the  $\sigma:\sigma_\alpha$  system, before simplification of the equations.

## S2.4 Dynamics of the full model

We begin by specifying the dynamics of intracellular mRNA through a saturating Hill curve parametrised by an input concentration of some transcription factor, a half-maximal activation concentration, and a Hill coefficient<sup>1,4,6,8</sup>. The general form of the dynamical equations for an mRNA species  $M_i$  is therefore defined as

$$\frac{d[M_i]}{dt} = \alpha_{0,i} + \alpha_{1,i}H(P_x) - \gamma_{M_i}[M_i], \quad (\text{S1})$$

where  $[X]$  denotes the concentration of species  $X$ ,  $\gamma_i$  is the rate of mRNA degradation,  $\alpha_{0,i}$  and  $\alpha_{1,i}$  are the basal and maximal transcription rates and  $H(P_x)$  is an activation, or inhibition, of the promoter regulated by the transcription factor  $P_x$ . The function regulating activation of the promoter for transcription of mRNA species  $M_i$  by species  $P_x$  is

$$H(P_x) = H_+(P_x) = \frac{[P_x]^{n_x}}{K_x^{n_x} + [P_x]^{n_x}}, \quad (\text{S2})$$

and likewise for inactivation/inhibition by species  $P_x$

$$H(P_x) = H_-(P_x) = \frac{K_x^{n_x}}{K_x^{n_x} + [P_x]^{n_x}}, \quad (\text{S3})$$

where  $\alpha_i$  is the maximal transcription rate,  $K_x$  is the microscopic dissociation constant, and  $n_x$  is a Hill coefficient defining cooperativity of binding to the promoter (steepness of the response curve).

### mRNA dynamics

The transcription of sigma factor mRNA,  $M_\sigma$ , is determined by the activity of the pLux promoter, which is activated by the presence of AHL (whose concentration is  $[A]$ ):

$$\frac{d[M_\sigma]}{dt} = \alpha_{0,\sigma} + \frac{\alpha_{1,\sigma}[A]^{n_A}}{K_A^{n_A} + [A]^{n_A}} - \gamma_{M_\sigma}[M_\sigma]. \quad (\text{S4})$$

Transcription of the corresponding anti-sigma mRNA,  $M_{\sigma\alpha}$ , is determined by the intracellular concentration of IPTG,  $I$ :

$$\frac{d[M_{\sigma\alpha}]}{dt} = \alpha_{0,\sigma\alpha} + \frac{\alpha_{1,\sigma\alpha}[I]^{n_I}}{K_I^{n_I} + [I]^{n_I}} - \gamma_{M_{\sigma\alpha}}[M_{\sigma\alpha}]. \quad (\text{S5})$$

Finally, the transcription of GFP mRNA is driven by the p<sub>20\_992</sub> promoter (see<sup>11</sup>). This promoter's activity is regulated directly by our sigma factor:

$$\frac{d[M_{\text{GFP}}]}{dt} = \alpha_{0,\text{GFP}} + \frac{\alpha_{1,\text{GFP}}[\sigma_{\text{free}}]^{n_\sigma}}{K_\sigma^{n_\sigma} + [\sigma_{\text{free}}]^{n_\sigma}} - \gamma_{M_{\text{GFP}}}[M_{\text{GFP}}]. \quad (\text{S6})$$



## Translation dynamics

Dynamics of proteins in then system consist of linear first order reactions, similar to typical models in the literature (e.g.,<sup>4,6,8</sup>). We explicitly account for different kinetics for all interactions, by including these as unique parameters unless otherwise specified.

In general, a protein species  $P_i$  evolves in time relative to its corresponding mRNA quantity  $M_i$  as

$$\frac{d[P_i]}{dt} = k_i[M_i] - \gamma_{P_i}[P_i], \quad (\text{S7})$$

where  $k_i$  is the forward rate constant for translation of mRNA  $M_i$  to protein  $P_i$ , and  $\gamma_{P_i}$  is the rate of first order degradation of  $P_i$ . In our model, we consider enzymatic degradation through the ssrA tag mechanism to be the dominant form of degradation for all  $P_i$ ; for this, we later include an extra term for each species and therefore here  $\gamma_{P_i} = \gamma$  i.e., a constant dilution for all protein species (discussed in the next section).

## ssrA-tagged proteins: enzymatic degradation

In order to implement enzymatic degradation of ssrA tagged proteins (free and bound sigma and anti-sigma, GFP), we define a nonlinear ssrA tag based degradation rate,  $f(X)$ , of the form previously used in<sup>3,10,12</sup> among others:

$$f(X) = \frac{\gamma_D}{c_e + X}, \quad (\text{S8})$$

where  $\gamma_D$  (molecules min<sup>-1</sup>) defines the maximal rate of enzymatic degradation,  $c_e$  (molecules) defines the half-activation threshold of the degradation, and

$$X = [\sigma_{\text{free}}] + [\sigma_{\alpha,\text{free}}] + [\sigma:\sigma_{\alpha}] + [\text{GFP}] \quad (\text{S9})$$

is the total number of ssrA tagged proteins present in the system (Table S4). The value of  $X$  is dynamic and so this equation implements a form of competition between available ssrA tag selecting proteases, since there is physically a limit on the number of available proteases.

## Sigma factor

Dynamics of the concentration of the free sigma factor,  $\sigma_{\text{free}}$ , are defined as

$$\begin{aligned} \frac{d[\sigma_{\text{free}}]}{dt} = & \overbrace{k_{\sigma}[M_{\sigma}]}^{(i)} - \overbrace{k_{\sigma:\sigma_{\alpha}}^{+}[\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}]}^{(ii)} + \overbrace{k_{\sigma:\sigma_{\alpha}}^{-}[\sigma:\sigma_{\alpha}]}^{(iii)} \\ & - \underbrace{f(X)[\sigma_{\text{free}}]}_{(iv)} - \underbrace{\gamma[\sigma_{\text{free}}]}_{(v)}. \end{aligned} \quad (\text{S10})$$

The various components of Equation S10 represent a number of chemical interactions, as follows:

- (i) generation of  $\sigma_{\text{free}}$  with translation rate  $k_{\sigma}$ , governed by the concentration of its mRNA;
- (ii) protein-protein interaction (association) representing the loss of  $\sigma_{\text{free}}$  through reversible binding, at rate  $k_{\sigma:\sigma_{\alpha}}^{+}$ , with the anti-sigma  $\sigma_{\alpha,\text{free}}$  to form the complex  $\sigma:\sigma_{\alpha}$ ;
- (iii) increase in  $\sigma_{\text{free}}$  resulting from dissociation of the  $\sigma:\sigma_{\alpha}$  complex, with dissociation rate  $k_{\sigma:\sigma_{\alpha}}^{-}$ ;
- (iv) degradation of  $\sigma_{\text{free}}$  via ssrA tag mechanism;
- (v) dilution of  $\sigma_{\text{free}}$  through cell division.

### Anti-sigma

The dynamics of the free anti-sigma, denoted by  $\sigma_{\alpha,\text{free}}$ , following a similar process as those of the sigma, are defined in Equation S11 below:

$$\begin{aligned} \frac{d[\sigma_{\alpha,\text{free}}]}{dt} = & \overbrace{k_{\sigma_{\alpha}}[M_{\sigma_{\alpha}}]}^{(i)} - \overbrace{k_{\sigma:\sigma_{\alpha}}^{+}[\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}]}^{(ii)} + \overbrace{k_{\sigma:\sigma_{\alpha}}^{-}[\sigma:\sigma_{\alpha}]}^{(iii)} \\ & - \underbrace{f(X)[\sigma_{\alpha,\text{free}}]}_{(iv)} - \underbrace{\gamma[\sigma_{\alpha,\text{free}}]}_{(v)}. \end{aligned} \quad (\text{S11})$$

with

- (i) the translation of  $\sigma_{\alpha,\text{free}}$  from its mRNA, at rate  $k_{\sigma_{\alpha}}$ ;
- (ii) loss of  $\sigma_{\alpha,\text{free}}$  as it reversibly binds to  $\sigma_{\text{free}}$  with rate  $k_{\sigma:\sigma_{\alpha}}^{+}$ , forming the  $\sigma:\sigma_{\alpha}$  complex;
- (iii) increase in free  $\sigma_{\alpha,\text{free}}$  from dissociation of  $\sigma:\sigma_{\alpha}$  at rate  $k_{\sigma:\sigma_{\alpha}}^{-}$ ;
- (iv) degradation of  $\sigma_{\alpha,\text{free}}$  via ssrA tag mechanism;
- (v) dilution of  $\sigma_{\alpha,\text{free}}$  through cell division.

The term (iv) was initially specified in a modified form

$$\gamma_{\text{Flag}} f(X)[\sigma_{\alpha,\text{free}}],$$

where  $\gamma_{\text{Flag}} < 1$  represents a loss of enzymatic degradation efficiency resulting from the presence of the Flag tag. However preliminary model analysis, via both manual parameter adjustment and global optimisation, indicated that the system output did not conform to observed measurements on inclusion of the factor  $\gamma_{\text{Flag}}$ . This scaling was therefore not included in the final model.

### Sigma:anti-sigma complex

We denote the sigma:anti-sigma complex by  $\sigma:\sigma_{\alpha}$ . Its dynamics consist of the formation, dissociation, and degradation (independent of the individual

degradation of its constituents) of the complex, as follows:

$$\frac{d[\sigma:\sigma_\alpha]}{dt} = \underbrace{k_{\sigma:\sigma_\alpha}^+ [\sigma_{\text{free}}] [\sigma_{\alpha,\text{free}}]}_{(i)} - \underbrace{k_{\sigma:\sigma_\alpha}^- [\sigma:\sigma_\alpha]}_{(ii)} - \underbrace{f(X) [\sigma:\sigma_\alpha]}_{(iii)} - \underbrace{\gamma [\sigma:\sigma_\alpha]}_{(iv)}, \quad (\text{S12})$$

with

- (i) the interaction between  $\sigma$  and  $\sigma_\alpha$  with binding (association) at rate  $k_{\sigma:\sigma_\alpha}^+$ ;
- (ii) the dissociation, at rate  $k_{\sigma:\sigma_\alpha}^-$ , of the complex  $\sigma:\sigma_\alpha$ ;
- (iii) the enzymatic degradation of sigma/anti-sigma bound in complex;
- (iv) dilution of the complex concentration resulting from cell division.

## Output

Production of the GFP from its mRNA is straightforward:

$$\frac{d[P_{\text{GFP}}]}{dt} = k_{\text{GFP}} [M_{\text{GFP}}] - f(X) [P_{\text{GFP}}] - \gamma [P_{\text{GFP}}]. \quad (\text{S13})$$

The GFP is translated with rate  $k_{\text{GFP}}$  and diluted at rate  $\gamma$ . Furthermore, since it is also tagged for degradation, we include the enzymatic degradation term  $f(X)$ .

### S3 Model simplification

In order to reduce the number of parameters required for subsequent model identification and parameter fitting, we simplified the original model. Through initial simulations, we determined that it was necessary to include the full dynamics of the complex formation and dissociation as these would be slow relative to the dynamics of the rest of the system. However, the separate dynamics of the mRNA and proteins could be merged.

If we consider the dynamics of mRNA transcription as being fast compared to the translation process, we can extend the quasi-steady-state approximation to the mRNA (Equations S4–S6) and the protein dynamics (Equations S10, S11 and S13). We consider mRNA dynamics to effectively be at steady state so that

$$\begin{aligned}\frac{d[M_\sigma]}{dt} &\approx 0, \\ \frac{d[M_{\sigma\alpha}]}{dt} &\approx 0, \\ \frac{d[M_{\text{GFP}}]}{dt} &\approx 0.\end{aligned}$$

Consequently, from Equations S4–S6, we obtain the following relationships for the mRNA concentrations:

$$[M_{\text{GFP}}] = \frac{1}{\gamma_{M\text{GFP}}} \left( \alpha_{0,\text{GFP}} + \frac{\alpha_{1,\text{GFP}}[\sigma_{\text{free}}]^{n_\sigma}}{K_\sigma^{n_\sigma} + [\sigma_{\text{free}}]^{n_\sigma}} \right), \quad (\text{S14})$$

$$[M_\sigma] = \frac{1}{\gamma_{M\sigma}} \left( \alpha_{0,\sigma} + \frac{\alpha_{1,\sigma}[A]^{n_A}}{K_A^{n_A} + [A]^{n_A}} \right), \quad (\text{S15})$$

$$[M_{\sigma\alpha}] = \frac{1}{\gamma_{M\sigma\alpha}} \left( \alpha_{0,\sigma\alpha} + \frac{\alpha_{1,\sigma\alpha}[I]^{n_I}}{K_I^{n_I} + [I]^{n_I}} \right). \quad (\text{S16})$$

We can substitute Equations S14–S16 into the relevant protein dynamics (Equations S10, S11 and S13) to obtain the following equations for the simplified GFP dynamics

$$\frac{d[P_{\text{GFP}}]}{dt} = \frac{k_{\text{GFP}}}{\gamma_{M\text{GFP}}} \left( \alpha_{0,\text{GFP}} + \frac{\alpha_{1,\text{GFP}}[\sigma_{\text{free}}]^{n_\sigma}}{K_\sigma^{n_\sigma} + [\sigma_{\text{free}}]^{n_\sigma}} \right) - f(X)[P_{\text{GFP}}] - \gamma_{P\text{GFP}}[P_{\text{GFP}}]. \quad (\text{S17})$$

The sigma and anti-sigma can be treated in a similar manner:

$$\begin{aligned}\frac{d[\sigma_{\text{free}}]}{dt} &= \frac{k_\sigma}{\gamma_{M\sigma}} \left( \alpha_{0,\sigma} + \frac{\alpha_{1,\sigma}[A]^{n_A}}{K_A^{n_A} + [A]^{n_A}} \right) - k_{\sigma:\sigma_\alpha}^+[\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}] \\ &\quad + k_{\sigma:\sigma_\alpha}^-[\sigma:\sigma_\alpha] - f(X)[\sigma_{\text{free}}] - \gamma_\sigma[\sigma_{\text{free}}], \quad (\text{S18})\end{aligned}$$

$$\begin{aligned} \frac{d[\sigma_{\alpha,\text{free}}]}{dt} = & \frac{k_{\sigma\alpha}}{\gamma_{M\sigma\alpha}} \left( \alpha_{0,\sigma\alpha} + \frac{\alpha_{1,\sigma\alpha}[I]^{n_I}}{K_I^{n_I} + [I]^{n_I}} \right) - k_{\sigma:\sigma\alpha}^+ [\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}] \\ & + k_{\sigma:\sigma\alpha}^- [\sigma:\sigma_\alpha] - f(X)[\sigma_{\alpha,\text{free}}] - \gamma_{\sigma\alpha}[\sigma_{\alpha,\text{free}}]. \end{aligned} \quad (\text{S19})$$

Finally, in order to aid the identification process, we combine parameters related to transcription, translation, and mRNA degradation by introducing the variables

$$\begin{aligned} \chi_{0,i} &= \frac{k_i \alpha_{0,i}}{\gamma_{Mi}} \\ \chi_{1,i} &= \frac{k_i \alpha_{1,i}}{\gamma_{Mi}}, \end{aligned}$$

where the subscript  $i$  indicates a particular chemical species (GFP,  $\sigma$ , and  $\sigma_\alpha$ ). Finally, introducing the lumped parameters  $\chi_{0,i}$  and  $\chi_i$  into Equations S17, S18 and S19, gives us the final form of the protein dynamics as follows:

$$\frac{d[P_{\text{GFP}}]}{dt} = \chi_{0,\text{GFP}} + \frac{\chi_{1,\text{GFP}}[\sigma_{\text{free}}]^{n_\sigma}}{K_\sigma^{n_\sigma} + [\sigma_{\text{free}}]^{n_\sigma}} - f(X)[P_{\text{GFP}}] - \gamma_{P\text{GFP}}[P_{\text{GFP}}], \quad (\text{S20})$$

$$\begin{aligned} \frac{d[\sigma_{\text{free}}]}{dt} = & \chi_{0,\sigma} + \frac{\chi_{1,\sigma}[A]^{n_A}}{K_A^{n_A} + [A]^{n_A}} - k_{\sigma:\sigma\alpha}^+ [\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}] \\ & + k_{\sigma:\sigma\alpha}^- [\sigma:\sigma_\alpha] - f(X)[\sigma_{\text{free}}] - \gamma_\sigma[\sigma_{\text{free}}], \end{aligned} \quad (\text{S21})$$

$$\begin{aligned} \frac{d[\sigma_{\alpha,\text{free}}]}{dt} = & \chi_{0,\sigma\alpha} + \frac{\chi_{1,\sigma\alpha}[I]^{n_I}}{K_I^{n_I} + [I]^{n_I}} - k_{\sigma:\sigma\alpha}^+ [\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}] \\ & + k_{\sigma:\sigma\alpha}^- [\sigma:\sigma_\alpha] - f(X)[\sigma_{\alpha,\text{free}}] - \gamma_{\sigma\alpha}[\sigma_{\alpha,\text{free}}], \end{aligned} \quad (\text{S22})$$

and complex dynamics

$$\frac{d[\sigma:\sigma_\alpha]}{dt} = k_{\sigma:\sigma\alpha}^+ [\sigma_{\text{free}}][\sigma_{\alpha,\text{free}}] - k_{\sigma:\sigma\alpha}^- [\sigma:\sigma_\alpha] - f(X)[\sigma:\sigma_\alpha] - \gamma[\sigma:\sigma_\alpha]. \quad (\text{S23})$$

Equations S20–S23 comprise the fully simplified system used for parameter identification and system simulation.

## S4 Parameter identification

Based on the wide-range characterisation of the reference comparator module (steady-state data indicated in the main material, Figure 1-B), we performed an identification to obtain an estimate of our unknown parameters. MATLAB 2016b<sup>1</sup> was used for all simulations.

The identification of unknown parameters was performed using the particle swarm optimisation routine built into MATLAB. This optimisation algorithm is a population-based technique, similar to a genetic algorithm, that attempts to find a globally optimal solution to an optimisation problem, therefore generating a set of parameters that are the most likely to correspond to the best possible solution for a given set of problem constraints.

Particle swarm optimization experiments were performed only on the data obtained at the 4-hour time point, as at this point we determined the experimental system had converged to a steady state response. Prior to optimisation, the data were filtered using a 2D Gaussian filter (MATLAB's `imgaussfit()` routine, with  $\sigma = 1.0$ ); the full range of points obtained across all IPTG and AHL inputs were used for the fitting. The objective function used for global optimisation computed the sum of squared errors between all model and data points, where each point's contribution to the total objective error was weighted according to the experimentally measured S.E.M. at that point (Supplementary Table S2).

We performed an exhaustive set of optimization experiments by changing weights in the objective function, selecting those parameter values that were observed heuristically to better match the time lapse experimental data.

Final values of optimised parameters obtained from the wide-range characterisation are shown in Table S6.

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<sup>1</sup>MATLAB Release 2016b, The MathWorks, Inc., Natick, Massachusetts, United States.

Parameter	Description	Baseline value	Optimised value	Source (baseline)
$\chi_{0,i}$	Basal transcription rate for mRNA species $i$ .	54 molecules min <sup>-1</sup>		12
$\chi_{1,i}$	Maximal transcription rate for mRNA species $i$ .	1080 molecules min <sup>-1</sup>		12
$\gamma_{Mi}$	Rate of decay of mRNA species $i$ .	0.54 min <sup>-1</sup>		12
$\gamma_{M_{GFP}}$	Rate of decay of GFP mRNA.	0.288 min <sup>-1</sup>		7
$K_{\sigma}$ **	Microscopic dissociation constant for $\sigma$ -regulated promoter.	3000 molecules	$1.98 \times 10^4$ molecules	*
$K_I$ **	Microscopic dissociation constant for IPTG-regulated promoter.	35 $\mu$ M	90 $\mu$ M	12
$K_A$	Microscopic dissociation constant for AHL-regulated promoter.	10 nM		2
$n_{\sigma}$ **	Hill coefficient for $\sigma$ -regulated promoter.	2	1.93	1
$n_A$ **	Hill coefficient for AHL-regulated promoter.	2	0.31	1
$n_I$ **	Hill coefficient for IPTG-regulated promoter.	2	0.46	1
$k_i$	Rate of translation of protein species $i$ from its mRNA.	81 min <sup>-1</sup>		12
$k_{GFP}$	Total rate of translation and maturation of GFP.	1.797 min <sup>-1</sup>		7
$\gamma_{Pi}$	Rate of dilution of protein species $i$ .	0.0277 min <sup>-1</sup>		25 min generation time.
$\gamma_D$ **	Maximal rate of degradation through ssrA tags.	1080 molecules min <sup>-1</sup>	701.2 molecules min <sup>-1</sup>	12
$c_e$ **	Half-maximal concentration for kinetics of ssrA tag based degradation.	0.1 molecules	0.01 molecules	12
$k_{\sigma:\sigma_{\alpha}}^+$	Rate of complex formation (association of $\sigma$ and $\sigma_{\alpha}$ )	0.018 min <sup>-1</sup> molecules <sup>-1</sup>		12
$k_{\sigma:\sigma_{\alpha}}^-$	Rate of complex dissociation ( $\sigma:\sigma_{\alpha}$ into its constituents)	0.00018 min <sup>-1</sup>		12

Supplementary Table S6: Kinetic parameters of the GRN model and their meaning, along with baseline values (used where a parameter was not being optimised/fitted to experimental data), the final optimised value after identification procedure described in Section S4 and sources for the baseline parameter values. N.b., the units for  $K_A$  and  $K_I$  are specified as  $\mu$ M corresponding to the experimental AHL and IPTG input signals used in the study; in contrast  $K_{\sigma}$  is specified in units of molecules since this promoter is directly activated by  $\sigma$ , a state variable that is itself specified in units of molecules. All parameter values are quoted as essentially being ‘per cell’, on average, in our aggregate model i.e., rates implicitly incorporate an assumed effect of a plasmid copy number  $\gg 1$ .

\*The initial value of  $K_{\sigma}$  was estimated via a preliminary model identification on the data presented in<sup>11</sup>.

\*\*This subset of the parameters were optimised according to the identification procedure described in Section S4.

## S5 Cellular consortium and comparator model

In order to validate the performance of our signal computation module, as well as test the fitted parameters (Table S6), we integrated it into a more complex system. This ‘extended model’ consisted of the proposed signal computation module acting as a reference comparator in a negative feedback control loop on a target population of cells, as proposed in<sup>5</sup>.

The target cells’ GRN was modelled as follows:

$$\frac{d[C]}{dt} = \chi_{0,C} + \frac{\chi_{1,C}[Q_t]^{n_Q}}{K_Q^{n_Q} + [Q_t]^{n_Q}} - g(X)[C] - \gamma[C], \quad (\text{S24})$$

$$\frac{d[D]}{dt} = \chi_{0,D} + \frac{\chi_{1,D}K_C^{n_C}}{K_C^{n_C} + [C]^{n_C}} - g(X)[D] - \gamma[D], \quad (\text{S25})$$

where the parameters have the same meaning as those in the comparator GRN discussed in the previous sections: all  $\chi$  and  $\gamma$  take the same values as in the comparator GRN ( $\chi_{0,i} = 54 \text{ molecules min}^{-1}$ ;  $\chi_{1,i} = 1080 \text{ molecules min}^{-1}$ ;  $\gamma = 0.0277 \text{ min}^{-1}$ ), while the target GRN specific parameters  $n_Q = n_C = 2$ ,  $K_Q = 9 \text{ molecules}$ , and  $K_C = 900 \text{ molecules}$ . The nonlinear enzymatic degradation function  $g(X)$  is analogous to  $f(X)$  (Equation S8), and is defined as

$$g(X) = \frac{\gamma_D}{c_e + X}, \quad (\text{S26})$$

where in the target GRN

$$X = [C] + [D], \quad (\text{S27})$$

and where  $\gamma_D = 701.2 \text{ molecules min}^{-1}$  and  $c_e = 0.01 \text{ molecules}$  are defined as above in S26, with the same values as those used in the comparator GRN.

In the extended model, we consider only a single external input, the IPTG corresponding to our external reference signal. AHL ( $[A]$ ) that is sensed by the reference comparator is generated from the target GRN; likewise, the reference comparator generates an orthogonal quorum molecule  $[Q]$  that is sensed by the target population. The equations for dynamics of the AHL molecule  $[A]$  concentration, for the comparator GRN (subscript  $c$ ), the target GRN (subscript  $t$ ) and external to the cells (subscript  $e$ ), are defined as

$$\frac{d[A_c]}{dt} = \nu_A[P_{\text{GFP}}] + \eta([A_e] - [A_c]) - \gamma_{A,i}[A_c], \quad (\text{S28})$$

$$\frac{d[A_t]}{dt} = \eta([A_e] - [A_t]) - \gamma_{A,i}[A_t], \quad (\text{S29})$$

$$\frac{\partial[A_e]}{\partial t} = \eta([A_c] - [A_e]) + \eta([A_t] - [A_e]) - \gamma_{A,e}[A_e] + \Theta \nabla^2[A_e]; \quad (\text{S30})$$



Note that in this extension of the model, the static AHL input  $[A]$  to the comparator GRN (in Equation S18, for example) is replaced by the dynamical variable  $[A_c]$ . Likewise the dynamics of  $Q$ , the signalling pathway from the comparator to the target, are defined as:

$$\frac{d[Q_c]}{dt} = \eta([Q_e] - [Q_c]) - \gamma_{Q,i}[Q_c], \quad (\text{S31})$$

$$\frac{d[Q_t]}{dt} = \nu_Q[D] + \eta([Q_e] - [Q_t]) - \gamma_{Q,i}[Q_t], \quad (\text{S32})$$

$$\frac{\partial[Q_e]}{\partial t} = \eta([Q_c] - [Q_e]) + \eta([Q_t] - [Q_e]) - \gamma_{Q,e}[Q_e] + \Theta \nabla^2[Q_e]. \quad (\text{S33})$$

Finally, it is crucial to note that Equations (S30) and (S33) are PDEs describing the spatio-temporal dynamics of the concentrations of  $A$  and  $Q$  in the extra-cellular environment. Parameters for the internal and external signalling molecule dynamics were taken from<sup>5</sup>, and are summarised in Table S7. For the solution of the spatial dynamics (PDEs in Equations (S30) and (S33)), a distance of  $20\mu\text{m}$  was chosen between the points corresponding to comparator and target cells.

Parameter	Value	Description
$\nu_A, \nu_Q$	$0.05 \text{ min}^{-1}$	Synthesis rate of $A$ and $Q$
$\eta$	$2 \text{ min}^{-1}$	Cell wall diffusion rate of $A$ and $Q$
$\gamma_{A,i}, \gamma_{Q,i}$	$0.4 \text{ min}^{-1}$	Internal degradation of $A$ and $Q$
$\gamma_{A,e}, \gamma_{Q,e}$	$0.2 \text{ min}^{-1}$	External $A$ and $Q$ degradation
$\Theta$	$4.9e - 10 \text{ m}^2 \text{ sec}^{-1}$	external diffusion rate of $A$ and $Q$

Supplementary Table S7: Parameter values for signalling molecules.

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